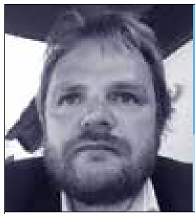

Rolf Poulsen

Basket Case

Making a case for Baskets. Let's talk Turkey.

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Armchair epidemiology abounds these days, so let me look at something more soothing in this month's column: Exchange rate volatility and what to do about it. In short, I make a case for baskets.

A peg to a basket

Throughout history, many countries have tried to peg or tie their currency to a larger, more dominant currency. Given what we know about the benefits of diversification, it seems natural to ask if improvement in some form can be archived by pegging to a diversified basket of currencies, for instance one that represents the trade composition of the country in question. This is what we now look at. And the answer is yes. For those of us with a quantitatvive finance background, that is quite unsurprisng. But politically, the issues we touch upon are highly sensitive. I won't go further into that, just trust me and my tin-foil hat. For that reason, the numerical example is a few years old.

The mathematics of basket pegging

A peg to a basket b of n currencies means that one unit of domestic currency pays b_1 units of foreign currency 1 and pays b_2 units of foreign currency 2 and … b_n units of foreign currency n. This means that the time t exchange rate for foreign currency i is

$$\frac{\text{peggedDOM}}{\text{xxx}_i}(t) = \left(\sum_{j=1}^{n} b_j \frac{\text{xxx}_i}{\text{xxx}_j}(t)\right)^{-1}, \qquad (1)$$

where I use the generic notation xxx_i for the ith foreign currency, DOM to denote domestic, and the (proper!) exchange rate notation where xxx/yyy means how many units of currency xxx is needed to buy one unit of currency yyy (i.e., European as opposed to US or UK notation). To understand Eq. (1), start looking at the jth term in the sum, say S, on the right-hand side; b_j is a number of units of currency j that is transferred into a number of units of currency i. The full sum S then gives the value of the basket in currency i. So one unit of pegged domestic currency is worth S units of foreign currency i, hence we quote the exchange rate as the reciprocal value of the sum. By having an explicitly stated time dependence in the basket, a peg with a drift, it is possible to do a controlled de- or re-valuation. This could help alleviate speculative pressure and volatility or jump risk.

Table 1: Turkish exchange rates and 2013 trade composition

	Eurozone EUR	Russia RUB	China CNY	USA USD	UK GBP	Switzerland CHF
TRYxxx on Jan. 2, 2012	2.44	0.0586	0.300	1.89	2.93	2.01
TRYxxx on June 1, 2012	2.30	0.0552	0.293	1.86	2.86	1.92
TRYxxx on Dec. 1, 2014	2.77	0.0437	0.361	2.22	3.48	2.30
Trade composition	0.45	0.17	0.15	0.10	0.08	0.05

Turkey as a worked example

Table 1 gives the Turkish (FX market abbreviation: TRY) exchange rate and trade weights for its six largest trading country counterpartners/currency markets. The trade weights have been calculated as the sum of import and export, with a specific counterpart relative to total import and export to the six counterparts. (These six represented just over half of Turkey's trade in 2013.)

On January 2, 2012 we could construct a basket that represents the trade weights and matches market exchange rates in this way:

$$b_i = \frac{\text{trade weight}_i}{\text{TRYxxx}_i(\text{Jan: }2, 2012)} = (0.1843, 2.9035, 0.5006, 0.0530; 0.0274, 0.0249).$$

On June 1, 2012 the pegged-to-basket TRYEUR exchange rate was

$$\frac{\text{peggedTRY}}{\text{EUR}}(\text{June 1; }2012) = \left(\sum_{J=1}^{n} b_j \frac{\text{EUR}}{\text{TRY}}(\ldots)\frac{\text{TRY}}{\text{xxx}_j}(\ldots)\right)^{-1} = 2.41.$$

Notice that when the basket is kept fixed over time (the number of units are the same), the values of the different currencies in the basket relative to the value of the whole basket change because exchange rates change. (In the same way that relative wealth shares in a buy-and-hold stock portfolio change over time.) To demonstrate, the vector of value weights on Dec. 1, 2014 is (0.465, 0.116, 0.169, 0.106, 0.089, 0.055). This means that when we describe the basket in the intuitively appealing weights way, there must always be an understanding of a reference date.

We can perform the same calculations for all trade counterparts over the period from early 2012 to late 2014. The results are shown in Figure 1, where the left-hand panel shows the behavior of the six currency crosses with and without pegging. To the naked eye the fluctuations of the pegged exchange rates are lower than for the actual exchange rates. This is quantified in the right-hand panel, where estimated volatilities (understood as the annualized standard deviation of daily logarithmic exchange rate changes) are depicted. We see that these are uniformly lower; a bit for the ruble, down to a third for the euro (which has a large weight in the basket), and about half for four other currencies. This shows that the Turkish lira has a large idiosyncratic (i.e. country-specific) volatility component.

Quantifying diversification gains and optimal pegs

Let $Z_i^b(k) = \ln$ denote daily ($dt = 1/252$ for daily observations) log-increments of the b-pegged exchange rate against currency i, i.e.

$$Z_i^b(k) = \ln\left(\frac{\text{peggedDOM/xxx}_i\,(dt * k)}{\text{peggedDOM/xxx}_i\,(dt * (k-1))}\right)$$

Assuming these Zs are independent over time, we can sensibly define Σ as the annualized covariance matrix:

$$Z^b_{i,j} = 252\text{cov}(Z^b_{i,} Z^b_j),$$

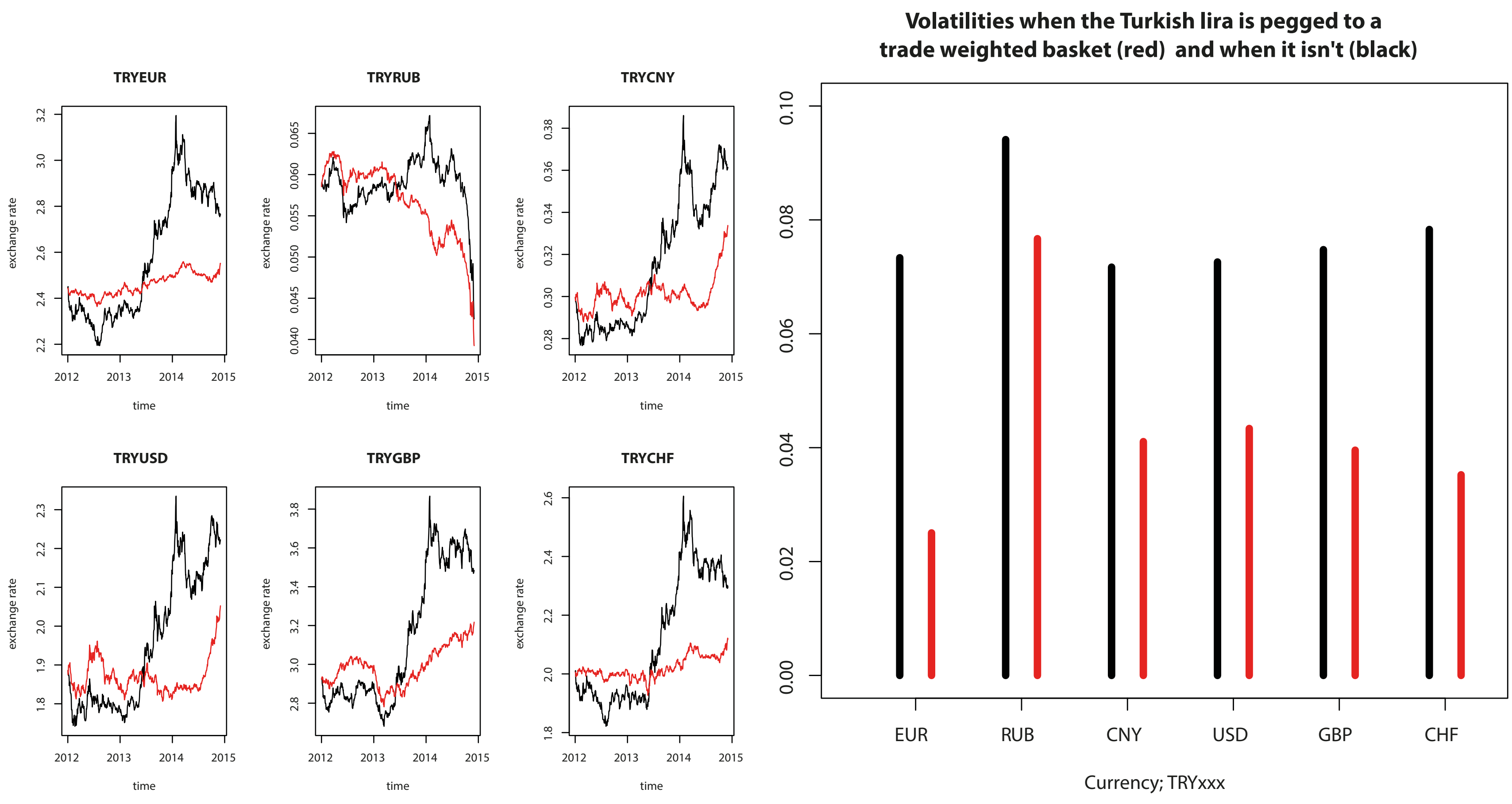
and estimate it by simple sample moments.

Given a vector w of trade weights, an immediate choice of basket b is the one that minimizes the variance of aggregated payments:

$$\min_b w^{\mathsf{T}} \Sigma^b w, \tag{2}$$

where the minimization is performed over baskets with positive entries scaled such that the initial values of pegged and actual exchanges are equal. The variance minimization approach takes a bird's eye view. Favorable movements in one currency can offset unfavorable movements in another; correlations matter. But that might not be a relevant or feasible view. The currency risk exposure comes from individual companies whose trade compositions do not match that of the whole economy. Hence another sensible

Figure 1: (a) shows the time-series behavior of the trade weight pegged (red) and unpegged (black) version of the Turkish lira against major trading partners; (b) shows the individual volatilities for pegged (red) and unpegged (black) versions of lira



measure of currency risk (towards a specific currency) would be how much it would cost to insure against unfortunate movements. This can be quantified through option prices. More model specifically, we do it by the Garman–Kohlhagen formula. The insurance cost/option price will depend on the ith diagonal element of the Σ^b matrix only. We ignore interest rates, and look at at-the-money options (meaning that put and call option prices are equal, or that insurance is bought against changes relative to current exchange rate) with 4-month maturities (a typical time period for companies' currency risk exposures). The overall basket-picking criterion then becomes minimizing the total insurance cost, i.e.

$$\min_b \sum_{i=1}^{n} |w_i| \text{Garman} - \text{Kohlhagen}(\ldots, \sqrt{\Sigma^b_{i,i}}), \tag{3}$$

Because short-term, at-the-money option prices are almost linear in volatility (σ, not σ^2), for all practical purposes solving (3) corresponds to minimizing trade-weighted average volatility.

Turkey again

As shown in Table 2, I have calculated the consequences of different basket choices (including "none"). We see the benefits of pegging; the insurance cost can be brought down from 1.77 percent of trade volume to 0.84 percent, and the trade price risk at the aggregate level all but disappears (standard deviation down from 6.38 to 0.21). But we also see that the choice of criterion (variance minimization or insurance cost minimization) matters. The variance-minimizing basket is very close to what trade weights give us, whereas insurance cost minimization is achieved with an almost perfect (98 percent) peg to the euro. When analyzing other countries, I find this "first past the post" effect (largest trading partner gets the full peg) to be a common phenomenon. So which criterion should be used? My knee-jerk reaction is that diversification is a strong force that we must be careful not to ignore, but in this case individual companies would still face currency risk even if the aggregate risk level is very low, and unlike for a stock market investor choosing his portfolio that could matter. Tricky stuff – as risk-sharing often is. A simple, quantitative argument for using the variance-minimizing portfolio is seen from the table: the variance-optimal basket performs quite well even if the criterion is insurance cost minimization – the converse much less so.

Table 2: Consequences of different Turkish lira basket compositions, with the baskets being expressed (in columns 2–7) in weight form with Jan. 2, 2012 as reference date

	EUR	RUB	CNY	USD	GBP	CHF	$100\sqrt{w^\top \Sigma^b w}$	Insurance cost (%)
No peg							6.38	1.77
EUR peg	1	0	0	0	0	0	2.63	0.85
USD peg	0	0	0	1	0	0	4.40	1.26
Trade weights	0.44	0.17	0.15	0.10	0.08	0.05	0.29	0.92
Trade variance min	0.43	0.20	0.15	0.09	0.08	0.06	0.21	0.93
Insurance cost min	0.98	0.01	0.01	0.00	0.00	0.00	2.52	0.84

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